A short history of Polish mathematics

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In this article I shall explain how Poland - a country having at the beginning of 20th century almost no mathematical traditions - could achieve, within a relatively short period 1919-1939, a good international position in such fields of mathematics as functional analysis, topology, set theory, functions of a real variable, logic and foundations of mathematics, and good forecasts for the future development of probability theory, differential equations and Fourier analysis.

The reason why we had poor mathematical traditions is rather clear. In 19th century – a period of a great development of mathematics in Western Europe – Poland was not an independent country. In 1795 it was partitioned between Austria, Russia and Prussia (Germany was not yet unified), and the independence was retrieved only at the end of 1918. Thus in 19th century the essential effort of the nation was set onto humanities, since literature and poetry were necessary for supporting the idea of independence and even for preservation of the language (in some periods the children in Prussian or Russian parts were taught only in German or Russian and Polish language was forbidden; the situation in the Austrian part was much more liberal). For that reason the mathematical papers were written only in Polish, so even if there were some original results, they remained not recognized by the mathematical community. Before the first World War (1914-18) there were in the Russian part of Polish territories two universities, a Russian University in Warsaw, offering lectures only in Russian (evacuated at the outburst of the war to Rostov-on Don, where it still exists; the previous University of Warsaw, with Polish professors, was closed after the upraising against Russians in 1863). Mathematics was taught there by a quite good Russian mathematician G. Voronoy, working mostly in number theory. The other university in Wilno (now Vilnius in Lithuania), was of much lower level. In the Prussian part of Poland, there was no university at all. Much better situation was in the Austrian part with two universities in Kraków (Cracow) and in Lwów (now Lviv in Ukraine). In the University of Kraków (founded in 1364 - the oldest in Poland, older than any German or Austrian University) there were at the beginning of 20th century two quite good Polish mathematicians: Kazimierz Żorawski (1866-1953), with Ph.D. made in Germany in Leipzig under Sophus Lie, and working in the differential geometry, and Stanisław Zaremba (1863-1942), who obtained his Ph.D. in Paris. Zaremba specialized in partial differential equations, potential theory and mathematical physics. Also in Lwów Univeristy there were two good mathematicians: Józef Puzyna (1856-1919) who specialized in complex analysis and integral equations, and Wacław Sierpiński (1882-1969). Sierpiński finished the Russian University in Warsaw in 1904 (under G. Voronoy), made his Ph.D. in 1906 in the University of Kraków and moved to Lwów, where he got a position at the University, first as a lecturer and since 1910 as a professor. Sierpiński at the beginning of his mathematical career worked in the classical number theory and later moved to the set theory (in 1909 he gave a university course of this theory, it seems to be first such a course ever given). After the outbreak of the first World War Russian troops took Lwów

and Sierpiński was internated first in Vyatka and later in Moscow, where he met several Russian mathematicians such as Egorov or Lusin. This meeting led to future collaboration of the Polish and Russian Schools of Mathematics. In turn, about the same time Warsaw was taken by the Prussian troops, and Prussians agreed to reopen in 1915 the Warsaw University. The chairs of mathematics were taken by two young mathematicians, Stefan Mazurkiewicz (1888-1945) and Zygmunt Janiszewski (1988-1920), both were topologists. Mazurkiewicz obtained his Ph.D. in Lwów under Sierpiński and Janiszewski in Paris in 1911. The board which passed his thesis consisted of H. Poincaré, H. Lebesgue and M. Fréchet. An important rôle in creation of the Polish Mathematical School was played by Zygmunt Janiszewski. Still before the end of the war it was clear that Poland will obtain independence, people were very entusiastic about it and made lot of plans for the future. Such a plan for mathematics was proposed by Janiszewski in his article "On the needs of mathematics in Poland". His main claim was that we have no chances in the well established theories, since we had no traditions and no great knowledge. However in the new fields,just emerging, we have the same chances as everybody else. Thus we should concentrate on the modern fields of mathematics (in this time it was set theory, topology, theory of functions of a real variable and logic with foundations of mathematics), and establish an international journal devoted only to these fields. The first volume of this journal, Fundamenta Mathematicae, appeared in 1920 and it was the first specialized mathematical journal in the world. Janiszewski could see this journal only in the galley-proofs, he died from an epidemic disease earlier in the same year at the age of only 32 years. After Poland obtained independence in November 1918, the Warsaw University had two more professors connected with mathematics: Jan Łukasieicz - a philosopher and logician (among his students were world famous mathematicians working in foundations of mathematics: Alfred Lindenbaum, Andrzej Mostowski and Alfred Tarski), and Sierpiński, who after internation in Russia did not return to Lwów, but to Warsaw – his home city. Many young talented students entered Warsaw University, the famous in the future mathematicians such as, Bronisław Knaster, Kazimierz Kuratowski, Szolem Mandelbrojt, Stanisław Saks, Alfred Tarski, Tadeusz Ważewski, Antoni Zygmund, and later Karol Borsuk, Samuel Eilenberg, Edward Szpilrajn (Marczewski), and many others. The Warsaw topologists (Sierpiński, Mazurkiewicz, Kuratowski, Knaster, later Borsuk and Eilenberg) had many important results in the theory of continua and curves, compacts and fix-point theory and in general topology (the system of axioms based upon the closure operation proposed by Kuratowski in his Ph.D. thesis and developed in [7] is now in common use). Widely known are Sierpiński's "carpet" and Borsuk's theory of retracts (applied later by Ważewski to differential equations). The term "Polish spaces", given by French mathematicians to complete separable metric spaces also illustrates the work of Polish topologists.

Important results were obtained also in the set theory. Sierpiński, Kuratowski, Tarski and others worked on basic questions of this theory, such as properties of ordinal numbers, the continuum hypothesis (see e.g [11]) and the axiom of choice. The Kuratowski's "maximum principle", rediscovered 13 years later by Max Zorn, is called now Zorn lemma or the Kuratowski-Zorm lemma.

The efforts of mathematicians working in real functions culminated later in famous books of Saks [10] and Zygmund [14].

The important feature of the Warsaw School was close cooperation of its participants. They discussed topics between themselves, presented them on the meetings of the Polish Mathematical Society and often published joint papers. Such a situation attracted to mathematics many young people, who also tried to obtain and publish new results even during the university studies. It was in contrast with the situation in Kraków, where Zaremba and Żorawski worked separately and no mathematical school was formed before the World War II.

The other, perhaps even more important mathematical school, was created in Lwów, due to Banach, Steinhaus with their students and collaborators. The story of Stefan Banach is quite extraordinary (for more details see Steihaus [12] and Kałuża [6] some details in these publications do not coincide). In 1916, when sitting in a Kraków park, Hugo Steinhaus overheard a conversation between two young people talking about the Lebesgue integral. Since the concept of this integral was quite fresh, Steinhaus approached to them and started a conversation. The young men were Stefan Banach, a student of the Lwów Polytechnic Institute (the studies were interrupted by the war) and Otto Nikodym (of the Radon-Nikodym theorem). In their talk Steinhaus mentioned the problem in Fourier series, he was working on in this time. It was already known that for a function f in $L^2(0, 2\pi)$ the partial sums $s_n(f)$ of its Fourier series are tending to f in the L^2-norm, and Steinhaus tried to work out whether a similar result is true for the space $L^1(0, 2\pi)$, i.e. whether for $f \in L^1(0, 2\pi)$ we have $\lim_n \int_0^{2\pi} [f(t) - s_n(f)(t)]dt = 0$. To his surprise, a few days later Banach constructed a counterexample, a function f in $L^1(0, 2\pi)$ such that $\lim_i ||f - s_{n_i}|| \to \infty$ for a certain sequence (n_i) of integers. Their joint paper was submitted to the Bulletin of Kraków Academy of Sciences and Letters and published, due to war, only in 1919. It was the first paper of Banach. Due to the war and lack of scientific information the authors did not know that the problem was already solved by Hahn. Since this time Steinhaus often discussed mathematics with Banach, Nikodym and their friend Wilkosz, who later became a professor of mathematics in the University of Kraków.

Stefan Banach was born on 30 March 1892. According to Steinhaus [12], a railroad employee, named Greczek brought a small baby to a Kraków laundress named Banach, gave her a certain amount of money and asked her to keep the baby. Greczek was father of Banach and the name of his mother is not known (perhaps it was a lady from aristocracy; my friend Marceli Stark told me once that he knew who was the Banach's mother, but that he promised never to tell it anybody), thus we should be rather speaking about Greczek spaces ! It was established later that the name of the laundress was different, and it seems that Banach was the name of a servant girl who agreed to testify that Banach was her son (in [6] it is stated that this girl was a real mother of Banach, what follows from suitable documents). When Banach was about 15 years old he had to care for himself getting money by tutoring school pupils in mathematics. In 1910 he finished the secondary school and (probably, the date is not sure) entered the Lwów Polytechnic Institute. At the outbreak of war in 1914 the studies were interrupted and Banach returned to Kraków. He has passed all necessary examinations for first two year of studies and it was the end of his formal education.

Banach dreamed about an assistentship in mathematics in his school – the Polytechnic Institute in Lwów, and thanks to Steinhaus it could be arranged. In 1920 Banach become

an assistant at the chair of mathematics occupied by professor Łomnicki. The same year he obtained his Ph.D. in spite of having no formal university studies. It was possible by a special permission from the Ministry of Education. His dissertation, translated to French, was published in Fundamenta Mathematicae ([1]). In this paper Banach gave axioms for the Banach spaces (the term itself was introduced later by M. Frechét and was not in use for a long time). In an article in the first volume of Studia Mathematica (1929) Steinhaus writes "Wir nennen – mit Herr Fréchet – einen Banachschen Raume, kurz einen B-Raum...". In his book [3], or its French version [4] Banach calls these spaces "spaces of type (B)", and the term " Banach space" is used directly only in 1940 in the mentioned below paper of M. Eidelheit. In his thesis Banach established basic properties of bounded linear operators between such spaces. There is also the Banach fix point theorem with an application to integral equations. Two years later Banach obtained his habilitation (published later in [3]) and in the same year obtained the position of an extraordinary professor in the University of Lwów. In his habilitation thesis Banach has solved a problem of Lebesgue. The question was whether one can construct on R^n a (positive) "measure" which is equal to one on the unit cube, is invariant under the isometric maps and is defined on all subsets of R^n. Of course, such a "measure", if existed, could not be countably additive, so that only the finite additivity was postulated. By an example of Hausdorff it was known that even such a measure cannot exist for $n \geq 3$, but the problem was open for $n = 1$ or 2. And Banach solved this problem in positive. Such a measure does exist on the line or plane. In the construction Banach used a developed later (see [4]) concept of generalized limit (based upon a preliminary version of the Banach-Hahn theorem). From this work emerged the concept of a Banach mean given later by John von Neumann. This work was completed by an interesting result obtained in a joint paper with Tarski [5]. Call two subsets A and B of R^n equivalent by means of a finite decomposition, if we can decompose both sets into a finite number of disjoint parts $A = \bigcup_1^k A_i, B = \bigcup_1^k B_i$ and the parts A_i and B_i are mutually isometric. Their result is as follows: Suppose that A and B are two bounded subsets of $R^n, n \geq 3$ with non-void interiors. Then these sets are equivalent by means of a finite decomposition. Usually this result is formulated for the case when A is a ball and B is the union of two disjoint balls of the same radius, but the result is much more general. In the proof the authors make use of a construction given earlier by Hausdorff, which showes, in particular, that for $n \geq 3$ the mentioned above problem of Lebesgue has a negative solution.

Banach quite soon attracted the attention of many good students and young mathematicians. They used to meet in a cafeteria and during long sessions discussed mathematics. In these meetings participated also the professors and sometimes the foreign guests. Soon formed a group of scientists working not only in functional analysis, the famous Lwów school of mathematics. It was closely connected with the Warsaw school, especially after 1927, when Kuratowski obtained a chair in Lwów Polytechnic Institute. Let me mention several names connected with this school. Stanisław Mazur, 1905-1981, was one of main collaborators of Banach. In particular he helped Banach in preparation the book [4], or its Polish version [3]. Mazur himself is known for numerous results in functional analysis (one of his classical results says that the convex envelope of compact subset of a Banach space is again compact) and summability theory, and widely known is the Mazur-Gelfand

theorem. Mazur also collaborated closely with Władysław Orlicz (1903-1990) obtaing results in non-linear functional analysis (polynomial operators) and in locally convex spaces (B_o-spaces, but their results on these spaces were published only after World War II). Orlicz is known for his results on orthogonal series, real functions, summability theory and functional analysis (polynomial operators, interpolation of operators and theory of modular spaces). After his name comes the concept of Orlicz Spaces. Very important mathematician in this group was Julian Schauder, 1899-1943. With his name there are connected such concepts and results as a Schauder basis, Schauder fix-point theorem and Lerey-Schauder theory of non-linear completely continuous operators (with topological methods based upon the concept of an index and with applications to partial differential equations). Herman Auerbach, (perished in 1942), is known for his results in the theory of convex bodies. He had an encyclopedic memory and knew lot of mathematical literature. So whenever Banach gave a subject to a student he was sending him to Auerbach for suitable references. Max (Meier) Eidelheit had only a few results before his premature death in 1943. Let me mention one of them, published in Studia Math. in 1940. Let X and Y be two Banach spaces and suppose that the algebras $L(X)$ and $L(Y)$ of bounded operators are algebraically isomorphic. Then the spaces X and Y are topologically isomorphic. It is one of first results in so called automatic continuity. One should mention also Stefan Kaczmarz (perished in 1939), working in the theory of orthogonal series (he published with Stenhaus a book in this subject), and much younger Stanisław Ulam, 1909-1984, the first Ph.D. of Kuratowski, and Marek (Mark) Kac (1914-1984) - with his Ph.D. made under Steinhaus. Both of them emigrated before 1939 to the United States and played there an important rôle (Ulam participated in the Los Alamos team working in construction of the nuclear weapon).

In 1928 Banach and Steinhaus founded in Lwów a new international journal "Studia Mathematica". The first volume appeared in 1929 and contained 14 papers. Four of them (by Kaczmarz, Steinhaus and two papers of Orlicz) were devoted to orthogonal series – which shows the current interest of the Lwów School in late tweenties. This volume contains also the Hahn-Banach theorem (in the second volume of Studia Math. Banach stated that a similar result was published by Hahn in 1927), the Banach's theorem on the continuity of an inverse operator, and the result of Mazur stating that all spaces $L^p(0,1), p \geq 1$, are mutually homeomorhic.

In 1931 the famous series "Monografie Matematyczne" (Mathematical Monographs, shortly M.M.) started with the classical book of Banach [4]. Many results, such as the closed graph theorem, the continuity of inverse of a linear operator, and the Banach-Steinhaus theorem were obtained in this book in a more general setting of an F-space, i.e. a completely metrizable topological vector space (Banach used this letter to honour M. Fréchet, and in this sense this term is used by the Polish School; later French mathematicians added the condition of local convexity and call such spaces Fréchet spaces). Other books in this series are positions [7], [10] and [11] on the reference list, as well as the mentioned above book of Kaczmarz and Steinhaus.

As it was mentioned earlier, the Lwów mathematicians almost daily gathered in a cafeteria, discussed mathematics and proved theorems. The cafeteria was called Kawiarnia Szkocka (Scottish Café). The sessions could last there for several hours ending late in the

night or even early morning (for a detailed description see [13] and [9]). During one night they have obtained a particularly interesting result. However, in the morning all their notes were wipped out from a marble table (they used pencils), and they could not reconstruct neither the proof nor even the result itself. Consequently Mrs Banach bought a copybook, gave it to a waiter and asked to hand it to the mathematicians whenever they appeared. In this way arose the famous Scottish Book. The first entry in this book is done by Stefan Banach on 17 July 1935 and last, No. 193, by Hugo Steinhaus on 31 May 1941. The mathematicians used to write there problems and sometime solutions. Usually for solving problems there were offered some prizes. For instance Mazur offered a live goose for solution of an approximation problem (known to be equivalent with the problem of existence of a Schauder basis in every Banach space). The goose was won by Per Enflo only in 1972 for the negative solution of this problem. The cafeteria was visited also by foreign guests (some entries in the Scottish Book are given by M. Fréchet, J. von Neumann, C. Offord and others).

The future plans of the Polish School were quite ambitious. It follows from the title of the Polish version of the Banach's book (see [3]), as well as from some papers of Mazur and Orlicz, and of Lerey and Schauder, that they planned to develop a theory of non-linear operators within functional analysis. The Polish Mathematical Society planned to organize two mathematical institutes, one in Lwów – devoted to applied mathematics, and one in Warsaw – devoted to the pure mathematics. Polish mathematicians realized that their mathematics is quite one-sided and rather narrow. In particular some classical theories, as the theory of analytic functions or algebra almost did not exist in Poland. They wanted to develop these theories too and also call more attention to applications of mathematics. There were some possibilities for developping more classical analysis. In 1930 Antoni Zygmund obtained a chair of mathematics in the University of Wilno, and soon got a very gifted student Józef Marcinkiewicz (1910-1940). Marcinkiewicz finished university in 1933, got his Ph.D. under Zygmund in 1935 and habilitated in 1937. In 1935/36 he stayed in Lwów working with Kaczmarz and Schauder and in 1939 was invited for a chair of mathematics to the Poznań University (founded in 1919), where, since 1937, Władysław Orlicz already was a professor. In 1939 Marcinkiewicz had already many published papers in real functions (singular integrals, differentiability of integrals, interpolation polynomials), trigonometric series, orthogonal systems, complex analysis, probability theory and theory of operators (interpolation theorems). At the outbreak of the World War II Marcinkiewicz was in Great Britain, working with Lttlewood, but immediately came back to Poland, served in the army, was taken prizoner of war by Sovjets (on the 1st of September 1939 Poland was invaded by Germany and, according to the Ribbentrop-Molotov agreement, on 17 of September by Sovjet Union, so that soon lost its independence and was partitioned again between Germany and Sovjet Union), and executed (probably in Katyń) together with several thousands of other Polish officers and civil servants. Antoni Zygmund writes that the death of Marcinkiewicz was the heaviest individual lost of Polish Mathematics during the second world was, he told me once that if Marcinkiewicz could live longer he would be one of greatest mathematicians of 20th century, perhaps better than Stefan Banach. In spite of a young age the list of his publications is about the same size as the list of Banach's publications, also his influence on future work in mathematical analysis is quite

substantial.

In the German part of Poland (it included Warsaw, Kraków and Poznań) not only Universities, but also secondary schools were closed. Poland was supposed to provide Germany with an unskilled cheap labour of uneducated people. Since Germany attacked first, many mathematicians moved eastward, and such scientists as Knaster, Orlicz, Saks and Szpilrajn (Marczewski) arrived to Lwów. The Sovjets took eastern part of Poland with Lwów and Wilno. They handed Wilno to Lithuania, which was for a while an independent country, but soon joined the Sovjet Union. Thanks to this independence Antoni Zygmund could emigrate to the United States, together with his family, and developped there later a great school of analysis. Lwów was incorporated to Ukraine – a part of Sovjet Union - but the University still worked with some Sovjet sciencists added. Banach even became a dean and all Polish mathematicians got there positions, including those who escaped from the German part of Poland. Also a volume of Studia Mathematica (almost ready for publication before the war) was published in 1940; only the Ukrainian abstracts had to be added. The meetings in Scottish Café continued (with a small break caused by the war) and new problems appeared (some of them were given by P.S. Alexandrov and S.L. Sobolev). In June 1941 Germany attacked Sovjet Union and took both Wilno and Lwów. All openly done scientific and educational work stopped in all of Poland. However some clandestine education was organized, mostly in Warsaw and in Kraków. It was very dangerous both for pupils and teachers, but even in such circumstances some Ph.D. degrees were granted).

Poland lost during the second world war about 50% of its mathematicians by death or emigration. Still before the war some mathematicians, as Kac, Ulam, Tarski and an excellent statistician Jerzy Spława-Neyman emigrated. The main reason was a shortage of positions. Before the World War II Poland had only 23 chairs of mathematics and 27 auxiliary positions, so that even such mathematicians as Nikodym or Orlicz had to teach in secondary schools. The reconstructed in 1945 year volume of Fundamenta Mathematicae (ready for print at the beginning of the war) contains a list of 22 names of mathematicians who perished during the war (mostly in the concentration camps). This list contains such names as Auerbach, Kaczmarz, Lindenbaum, Łomnicki, Marcinkiewicz, Rajchman, Saks, Schauder and Schreier. Also Banach and Mazurkiewicz passed away just after the end of war. In this situation the task of restoration of mathematics in Poland was very hard. We lost not only many great mathematicians, but also one generation of youg people, who either perished during the war, or were not sufficiently educated. Consequetly there was a serious generation gap after the war. There were older mathematicians like Sierpiński, Kuratowski, Borsuk and Mazur and younger collegues about thirty years old, while the intermediate generation consisted only with a few mathematicians like Mostowski, Marczewski (during the war he had to change his name from Szpilrajn) Mikusiński or Sikorski. Also a five years break of research was very harmful. Here the situation was quite different from that in Western Europe (I was very surprised when learning that, for instance, in France or Netherlands the mathematicians worked more or less normally and published their papers). One should add also the losses in literature (for instance, the mathematical library of the Warsaw University was burned off in 1939, not mentioning the lack of information on recent results). Also the change of borders, with Lwów University

moved to Wrocław (former German Breslau), and Wilno University moved to Toruń with all documentation and libraries left over, added to our loses. Nevertheless the job of reconstruction and augmentation of Polish mathematics was done, but it should be the subject of a different story.

References

[1] S. Banach, Sur les opérations dans les ensembles abstraits et leur application aux équations integrales, Fund. Math. 3 (1922), 133-181.
[2] — , Sur le problème de la mesure, Fund. Math. 4 (1923), 7-33.
[3] — , Theory of Operations. vol.I. Linear Operations (in Polish), Warszawa 1931.
[4] — , Théorie des Opérations Linéaires, M.M. vol. 1, Warszawa 1932.
[5] S. Banach and A Tarski, Sur la décomposition des ensembles de points en parties respectivement congruentes, Fund. Math. 6 (1924), 244-277.
[6] R. Kałuża, The Life of Stefan Banach, Birkhaüser, Boston 1996.
[7] K. Kuratowski, Topologie I, M.M. vol. 3, Warszawa 1933.
[8] — , A Half Century of Polish Mathematics, International series in pure and applied mathematics vol. 108, Pergamon Press and PWN, 1980.
[9] R. D. Mauldin (ed.), Mathematics from the Scottish Café, Birkhaüser 1981.
[10] S. Saks, Theory of the Integral, M.M. vol. 7, Warszawa 1937.
[11] W. Sierpiński, Hypothèse du Continu, M.M. vol. 4, Warszawa 1934.
[12] H. Steinhaus, Stefan Banach , Studia Math. Ser. Spec. Z.1. (1963), 7-15.
[13] S. Ulam, Adventures of a Mathematician, University of California Press, Berkeley 1976.
[14] A. Zygmund, Trigonometrical Series, M.M. vol. 5, Warszawa 1935.

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